

QUESTION Using the usual expansion for  $\sin(A+B)$  (how would you prove this for  $A, B$  complex?) prove that if  $z = x + iy$  then

$$\sin(x + iy) = \sin x \cosh y + i \cos x \sinh y.$$

ANSWER Usual expansion for  $\sin(a+b)$  is

$$\sin(a + b) = \sin A \cos B + \sin B \cos A$$

(This may be proved for complex  $A, B$  by expressing both sides in terms of the exponential function.) Now  $\cos iy = \cosh y$ ,  $\sin iy = i \sinh y$ , giving the result.