ELEC1011 Communications and Control
Coursework 1: Modulation

Introduction

This coursework contributes 5% of your mark for ELEC1011 Communications and Control. It is therefore worth half a credit and should take you up to 5 hours to complete it. Please do not spend any more time than this since I'm sure you have better things to be doing!

Besides discussing ideas with your classmates, you are required to work entirely on your own to produce a write-up, which only needs to include the figures, calculations and comments that are directly requested in the exercises below. Your write-up can contain a combination of hand-written work, computer printouts and pages from this document. When you are finished, you need to print out a receipt from C-BASS
`https://handin.ecs.soton.ac.uk/handin/0910/ELEC1011/1/`
You then need to staple your receipt together with your write-up and submit it at the ECS reception before 4pm on 17/03/2010.

Here are some tips that will help you get a high mark in this coursework:

- Include your workings for each calculation in your write-up.
- Each figure in your write-up should have well labelled and accurately annotated axes, as well as a relevant title and/or legend that identifies which signals are shown.
- If you draw your figures by hand, make sure you use a ruler.
- You may like to use Matlab to help you complete the exercises and to produce the figures. To help you with this, the appendix of this document provides Matlab code that will let you generate the signals shown in Figures 1a, 1b and 3a.

If you notice any mistakes in this document or have any queries about it, please email me at `rm@ecs.soton.ac.uk`

Have fun, Rob Maunder.

1 Sampling, Pulse Amplitude Modulation and Multiple Access

Figure 1a provides a time-domain plot of the signal $v_1(t)$, while a time-domain plot of the signal $v_2(t)$ is provided in Figure 1b. These signals are plotted

together in Figure 2a. Both signals have the same amplitude spectrum $|V_1(f)| = |V_2(f)|$, which is shown in the frequency-domain plot of Figure 2b. The signal $v_1(t)$ is to be sampled.

a) State the minimum sampling frequency that can be used to sample the signal $v_1(t)$ without introducing aliasing.

b) Draw vertical lines into Figure 1a to indicate each instant at which the signal $v_1(t)$ is sampled when using the minimum sampling frequency. Assume that the first sample is taken when $t = 0$ ms.

The samples are to be transmitted using Pulse Amplitude Modulation (PAM).

c) Draw a PAM representation of the signal $v_1(t)$ into Figure 1a, using a pulse duration of 10 ms.

Multiple access is to be achieved by multiplexing the PAM representation of the signal $v_1(t)$ with a PAM representation of the signal $v_2(t)$.

d) State the minimum sampling frequency that can be used to sample the signal $v_2(t)$ without introducing aliasing.

e) Draw vertical lines into Figure 1b to indicate each instant at which the signal $v_2(t)$ is sampled when using the minimum sampling frequency. Select the time instants that are *most appropriate* for allowing the PAM representation of $v_2(t)$ to be multiplexed with the one that you drew in Figure 1a for $v_1(t)$.

f) Draw a PAM representation of the signal $v_2(t)$ into Figure 1b, using a pulse duration of 10 ms.

g) Draw the multiplexed PAM representations of $v_1(t)$ and $v_2(t)$ into Figure 2a.

2 Sampling, Quantisation, Pulse Coded Modulation and Manchester Line Coding

Figure 3a provides a time-domain plot of the signal $v_3(t)$, while a frequency-domain plot of the corresponding amplitude spectrum $|V_3(f)|$ is provided in Figure 3b. The signal $v_3(t)$ is to be sampled.

a) State the minimum sampling frequency that can be used to sample the signal $v_3(t)$ without introducing aliasing.

b) Draw vertical lines into Figure 3a to indicate each instant at which the signal $v_3(t)$ is sampled when using the minimum sampling frequency. Assume that the first sample is taken when $t = 0$ ms.

The samples are to be quantised using the quantisation levels $\{-2.15, -1.34, -0.76, -0.25, 0.25, 0.76, 1.34, 2.15\}$.

c) Draw horizontal lines into Figure 3a to represent each of the quantisation levels. Draw crosses at the appropriate intersections of your horizontal and vertical lines to show which quantisation level is selected for each sample.

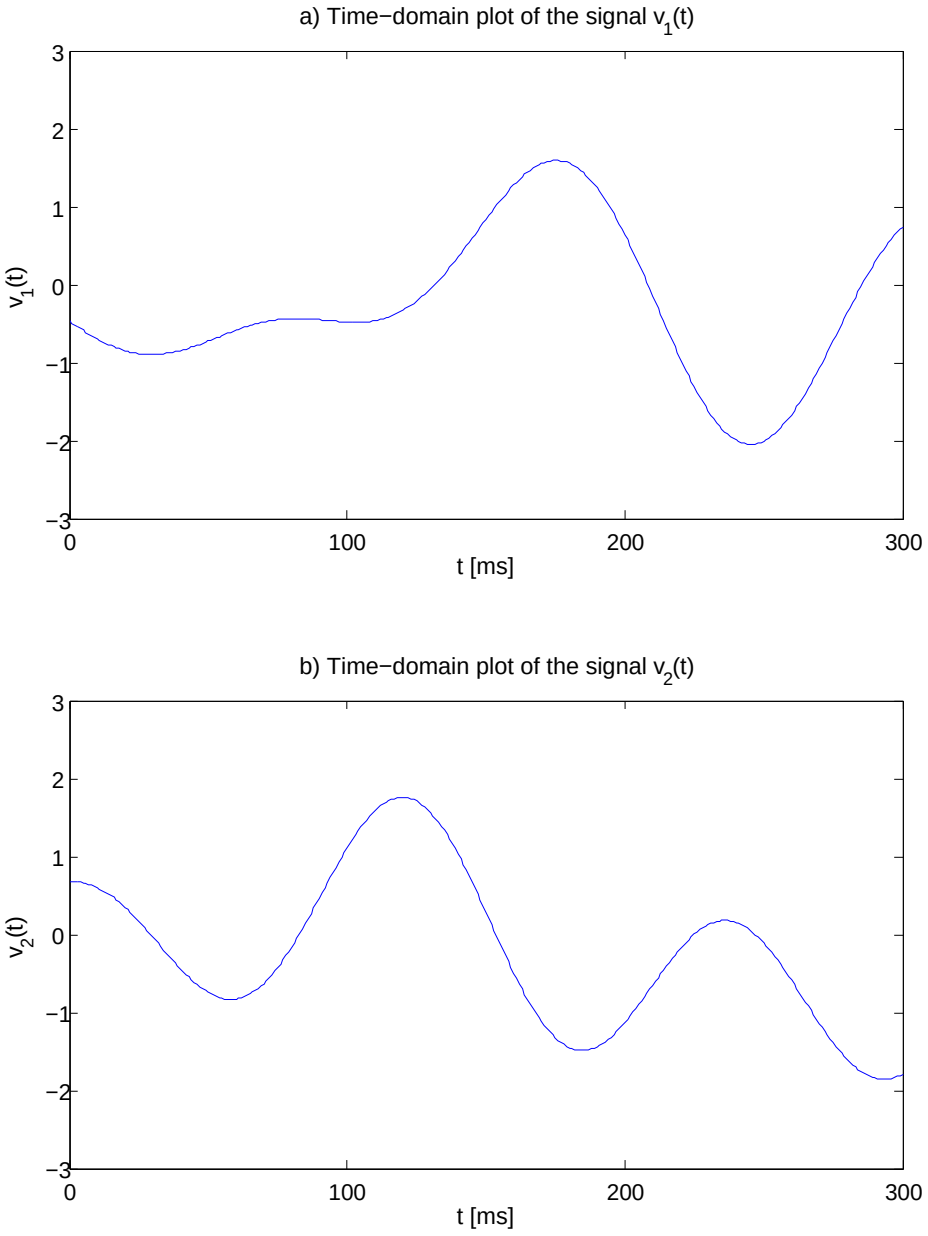


Figure 1: a) Time-domain plot of the signal $v_1(t)$. b) Time-domain plot of the signal $v_2(t)$.

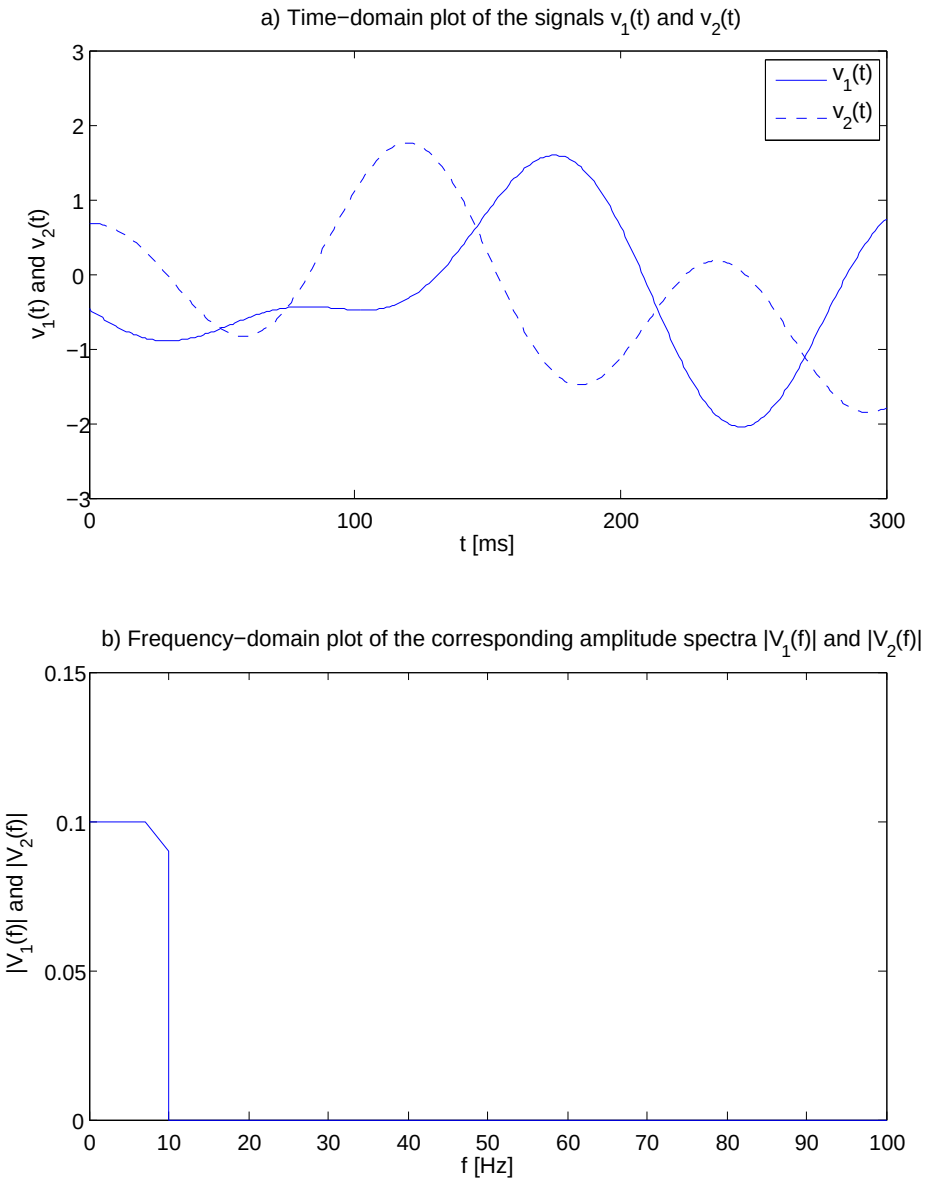


Figure 2: a) Time-domain plot of the signals $v_1(t)$ and $v_2(t)$. b) Frequency-domain plot of the corresponding amplitude spectra $|V_1(f)|$ and $|V_2(f)|$.

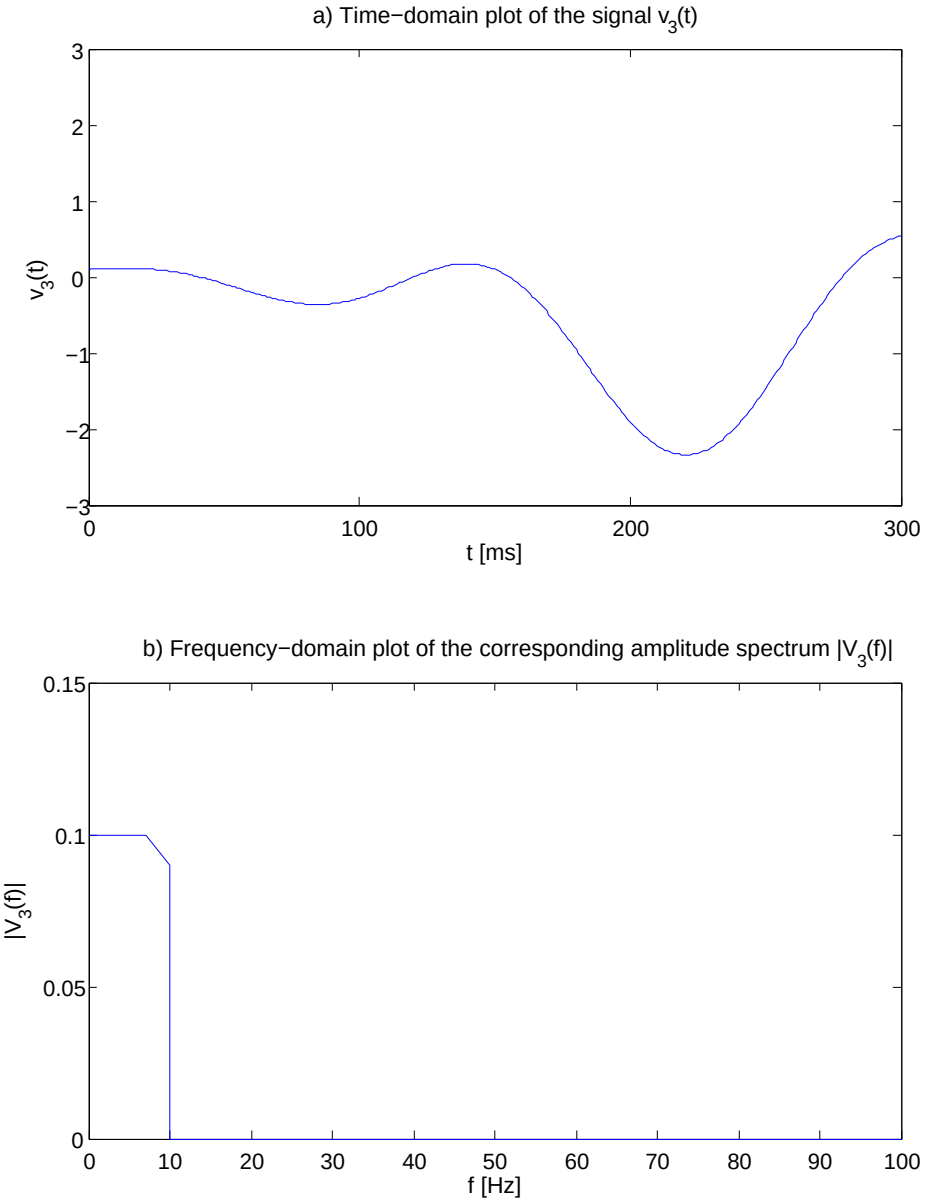


Figure 3: a) Time-domain plot of the signal $v_3(t)$. b) Frequency-domain plot of the corresponding amplitude spectrum $|V_3(f)|$.

d) State the advantage of using these non-uniform quantisation levels.

The quantised samples are to be encoded using Gray-coded Pulse Code Modulation (PCM).

e) State how many bits are required for the binary codewords used to represent each quantisation level.

f) State the bit rate that naturally results for this sampling, quantisation and PCM scheme.

g) Label each of your horizontal lines in Figure 3 with an appropriate Gray-coded binary codeword.

h) State the advantage of using Gray coding in this scheme.

i) State the bit sequence that results from the sampling, quantisation and PCM coding of the signal $v_3(t)$.

A Manchester line code is to be used to transmit the PCM-coded quantised samples.

j) Sketch a plot of the resultant Manchester line code signal, remembering to *accurately* annotate the time axis.

3 Amplitude Modulation

The signal $v_3(t)$ of Figure 3a is to be Amplitude Modulated (AM) onto a $f_c = 40$ Hz carrier.

a) Sketch a time-domain plot of the AM signal $v_{am}(t)$ that results when using a modulation sensitivity of $k_{am} = 2$ and a DC offset of $V_c = 5$, remembering to *accurately* annotate both axes.

b) Determine the modulation index for the AM signal $v_{am}(t)$.

c) Sketch a frequency-domain plot of the AM signal's amplitude spectrum $|V_{am}(f)|$, remembering to *accurately* annotate both axes.

d) State the bandwidth of the AM signal $v_{am}(t)$.

The signal $v_3(t)$ has a normalised average power of 1.

e) Determine the normalised average power of each sideband in the AM signal $v_{am}(t)$.

f) Determine the normalised average power of the carrier in the AM signal $v_{am}(t)$.

g) State the fraction of the power in the AM signal $v_{am}(t)$ that conveys information.

The AM signal $v_{am}(t)$ is to be demodulated.

h) State why the AM signal $v_{am}(t)$ can be demodulated using an envelope detector.

i) Name the two stages of an envelope detector.

j) For the case of demodulating your AM signal $v_{\text{am}}(t)$, sketch time-domain plots of the signals that are obtained after each stage of the envelope detector, remembering to *accurately* annotate both axes.

4 Double SideBand Suppressed Carrier Modulation and Quadrature Amplitude Modulation

Equation 1 provides an expression for the signal $v_4(t)$.

$$v_4(t) = 2 + \cos(10\pi t + \pi/4) + 3\cos(20\pi t - \pi) \tag{1}$$

a) Sketch a frequency-domain plot of the amplitude spectrum $|V_4(f)|$ for the signal $v_4(t)$, where $|V_4(f)| \geq 0$ *for all* f. Remember to *accurately* annotate both axes.

b) Sketch a frequency-domain plot of the phase spectrum $\angle V_4(f)$ for the signal $v_4(t)$, where $-\pi \leq \angle V_4(f) < \pi$ *for all* f. Remember to *accurately* annotate both axes.

The signal $v_4(t)$ of Equation 1 is to be Double SideBand Suppressed Carrier (DSBSC) modulated onto an *in-phase* cosine carrier having a frequency of $f_{\text{c}} = 40$ Hz.

c) Express the resultant DSBSC signal $v_{\text{dsb4}}(t)$ as a sum of *cosines* having *different* frequencies.

d) Sketch a frequency-domain plot of the corresponding amplitude spectrum $|V_{\text{dsb4}}(f)|$, where $|V_{\text{dsb4}}(f)| \geq 0$ *for all* f. Remember to *accurately* annotate both axes.

e) Sketch a frequency-domain plot of the corresponding phase spectrum $\angle V_{\text{dsb4}}(f)$, where $-\pi \leq \angle V_{\text{dsb4}}(f) < \pi$ *for all* f. Remember to *accurately* annotate both axes.

Equation 2 provides an expression for the signal $v_5(t)$.

$$v_5(t) = 3\cos(10\pi t + \pi/2) - 2\sin(15\pi t + \pi/4) \tag{2}$$

f) Sketch a frequency-domain plot of the amplitude spectrum $|V_5(f)|$ for the signal $v_5(t)$, where $|V_5(f)| \geq 0$ *for all* f. Remember to *accurately* annotate both axes.

g) Sketch a frequency-domain plot of the phase spectrum $\angle V_5(f)$ for the signal $v_5(t)$, where $-\pi \leq \angle V_5(f) < \pi$ *for all* f. Remember to *accurately* annotate both axes.

The signal $v_5(t)$ of Equation 2 is to be Double SideBand Suppressed Carrier (DSBSC) modulated onto a *quadrature-phase* sine carrier having a frequency of $f_{\text{c}} = 40$ Hz.

h) Express the resultant DSBSC signal $v_{\text{dsb5}}(t)$ as a sum of *cosines* having *different* frequencies.

i) Sketch a frequency-domain plot of the corresponding amplitude spectrum $|V_{\text{dsb5}}(f)|$, where $|V_{\text{dsb5}}(f)| \geq 0$ *for all* f. Remember to *accurately* annotate both axes.

j) Sketch a frequency-domain plot of the corresponding phase spectrum $\angle V_{\text{dsb5}}(f)$, where $-\pi \leq \angle V_{\text{dsb5}}(f) < \pi$ *for all* f. Remember to *accurately* annotate both axes.

The DSBSC signals $v_{\text{dsb4}}(t)$ and $v_{\text{dsb5}}(t)$ are to be combined to give a Quadrature Amplitude Modulation (QAM) signal $v_{\text{qam}}(t)$.

k) Express the resultant QAM signal $v_{\text{qam}}(t)$ as a sum of *cosines* having *different* frequencies. You will need to use phasors to combine consines having the same frequency.

l) Sketch a frequency-domain plot of the corresponding amplitude spectrum $|V_{\text{qam}}(f)|$, where $|V_{\text{qam}}(f)| \geq 0$ *for all* f. Remember to *accurately* annotate both axes.

m) Sketch a frequency-domain plot of the corresponding phase spectrum $\angle V_{\text{qam}}(f)$, where $-\pi \leq \angle V_{\text{qam}}(f) < \pi$ *for all* f. Remember to *accurately* annotate both axes.

The signal $v_4(t)$ is to be extracted from the QAM signal $v_{\text{qam}}(t)$ using coherent demodulation. The first step is to combine the QAM signal $v_{\text{qam}}(t)$ with an *in-phase* cosine carrier having a frequency of $f_{\text{c}} = 40$ Hz.

n) Express the resultant intermediate signal $v_{\text{o4}}(t)$ as a sum of *cosines* having *different* frequencies.

o) Sketch a frequency-domain plot of the corresponding amplitude spectrum $|V_{\text{o4}}(f)|$, where $|V_{\text{o4}}(f)| \geq 0$ *for all* f. Remember to *accurately* annotate both axes.

p) Sketch a frequency-domain plot of the corresponding phase spectrum $\angle V_{\text{o4}}(f)$, where $-\pi \leq \angle V_{\text{o4}}(f) < \pi$ *for all* f. Remember to *accurately* annotate both axes.

q) Name the second step in coherent detection and state how this reconstructs $v_4(t)$ from $v_{\text{o4}}(t)$.

5 M-ary Shift Keying

5.1 On-Off Keying

The bit sequence 001001011010 is to be On-Off Keying (OOK) modulated onto a $f_{\text{c}} = 40$ Hz carrier at a bit rate of $R_{\text{b}} = 40$ Hz. The constellation diagram for OOK is provided in Figure 4.

a) Determine the duration of the resultant OOK signal $v_{\text{ook}}(t)$.

a) Sketch a time-domain plot of the OOK signal $v_{\text{ook}}(t)$, remembering to annotate both axes.

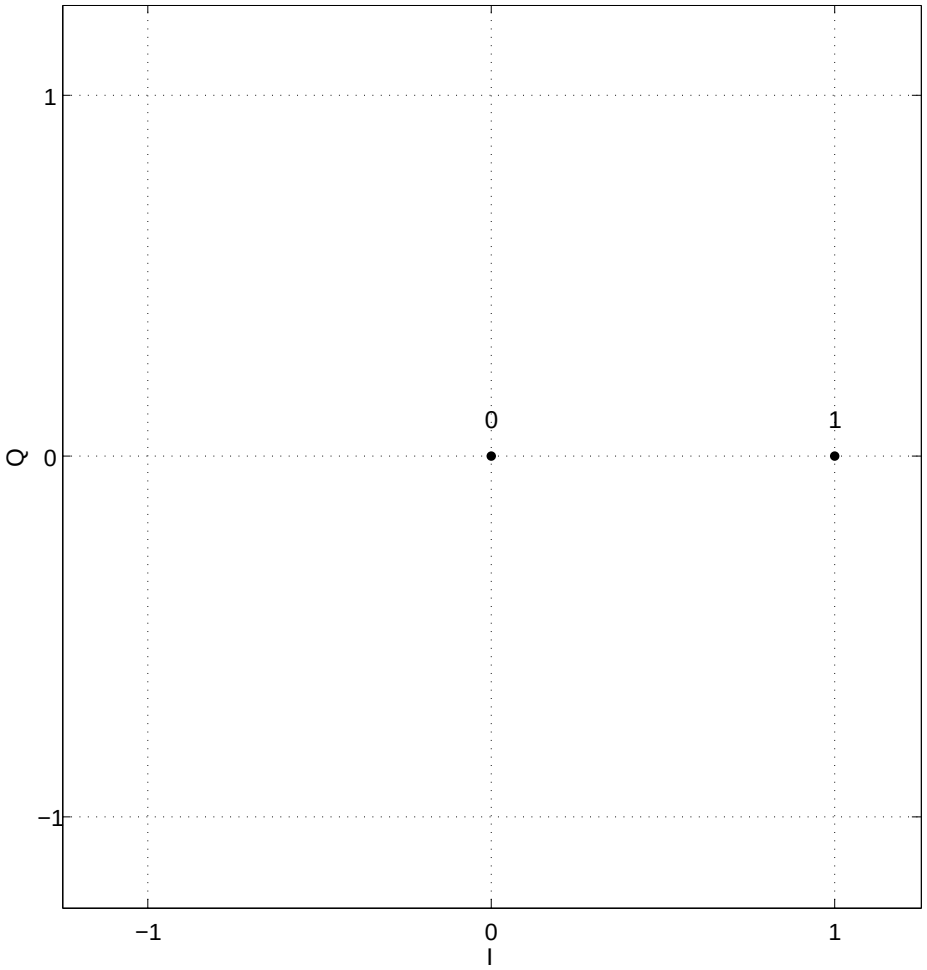


Figure 4: OOK constellation diagram.

b) State the bandwidth that is required for the OOK signal $v_{\text{ook}}(t)$, as well as the range of frequencies that it occupies.

c) Determine the bandwidth efficiency of the described OOK scheme.

5.2 $M = 4$-ary Frequency Shift Keying

A bit sequence is transmitted at a bit rate of $R_{\text{b}} = 40$ Hz using $M = 4$-ary Frequency Shift Keying (4FSK). A time-domain plot of the received 4FSK signal $v_{\text{4fsk}}(t)$ is shown in Figure 5.

a) Determine the symbol rate R_{s}.

b) Draw vertical lines into Figure 5 to identify the boundaries between consecutive symbol periods. Assume that the first symbol period begins at $t = 0$ ms.

c) Determine the minimum separation that is required between any pair of the $M = 4$ frequencies that are used by the 4FSK signal $v_{\text{4fsk}}(t)$.

The 4FSK signal $v_{\text{4fsk}}(t)$ uses the $M = 4$ frequencies f_1, f_2, f_3 and f_4, where $f_1 < f_2 < f_3 < f_4$ and $f_1 = 10$ Hz. The minimum possible separation between these frequencies is employed.

d) Determine values for f_2, f_3 and f_4. Write these values in Table 1.

e) State the bandwidth that is required for the FSK signal $v_{\text{4fsk}}(t)$, as well as the range of frequencies that it occupies.

f) Determine the bandwidth efficiency of the described OOK scheme.

The binary codeword that is conveyed by the activation of each frequency f_1, f_2, f_3 and f_4 is identified in Table 1.

g) Annotate each of the symbol periods that you identified in Figure 5 with the binary codeword that it conveys.

5.3 Digital Quadrature Amplitude Modulation

The bit sequence 001010011100 is to be modulated onto a $f_{\text{c}} = 40$ Hz carrier at a bit rate of $R_{\text{b}} = 40$ Hz and a symbol rate of $R_{\text{s}} = 20$ Hz. This is achieved by the Digital Quadrature Amplitude Modulation (digital-QAM) of the signal $v_6(t)$ onto an in-phase carrier and the signal $-v_7(t)$ onto a quadrature-phase carrier, where time-domain plots of the signals $v_6(t)$ and $v_7(t)$ are provided in Figure 6a and 6b, respectively.

a) Determine the number of bits per symbol k.

b) Determine the number of constellation points M.

c) Draw a constellation diagram for this digital-QAM scheme, remembering to *accurately* annotate both axes and to label each constellation point with the corresponding bit sequence.

d) Name the type of coding and modulation you have drawn in your constellation diagram.

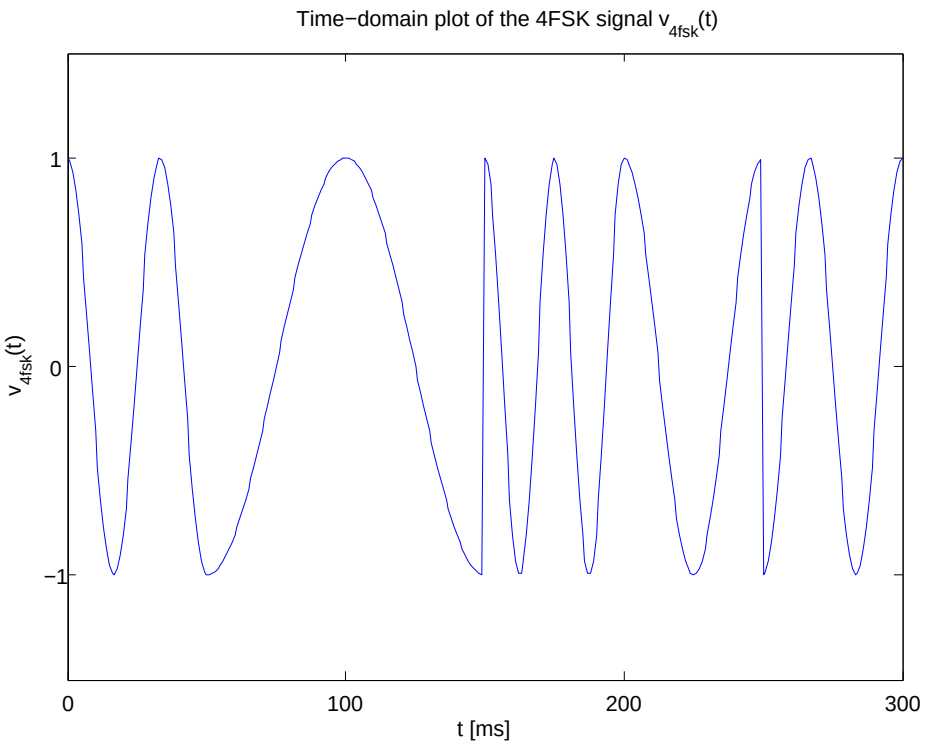


Figure 5: Time-domain plot of the 4FSK signal $v_{4\text{fsk}}(t)$.

i	f_i	codeword
1	10 Hz	00
2		01
3		10
4		11

Table 1: Mapping between frequencies and bit sequences in the 4FSK signal of Figure 5.

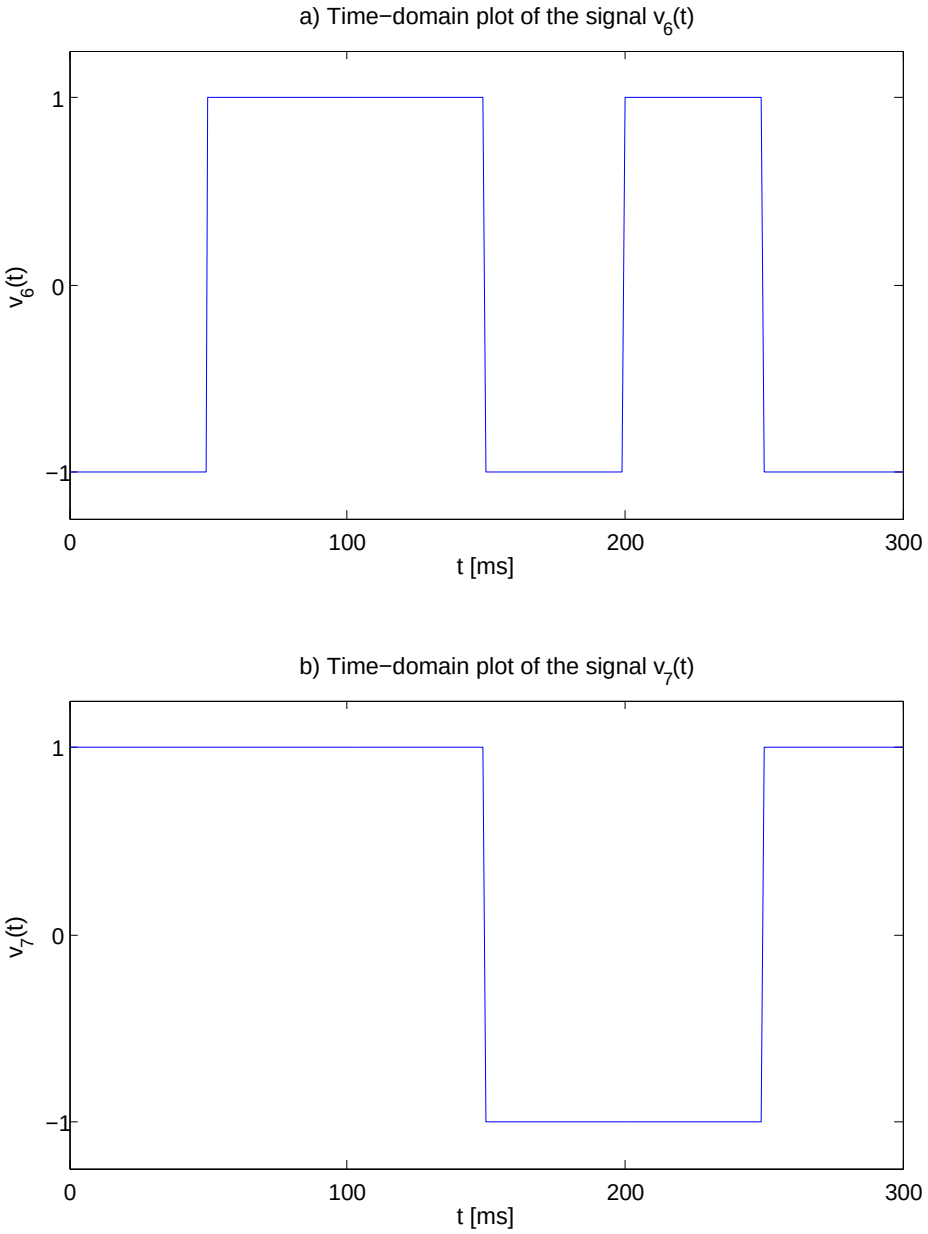


Figure 6: a) Time-domain plot of the signal $v_6(t)$. b) Time-domain plot of the signal $v_7(t)$.

e) State the bandwidth that is required for the digital-QAM signal, as well as the range of frequencies that it occupies.

f) Determine the bandwidth efficiency of the described digital-QAM scheme.

5.4 $M = 8$-ary Phase Shift Keying

The bit sequence 0000101011001100000 is to be modulated onto a $f_c = 40$ Hz carrier at a bit rate of $R_b = 60$ Hz, using the $M = 8$-ary Phase Shift Keying (8PSK) scheme described by the constellation diagram of Figure 7. This is to

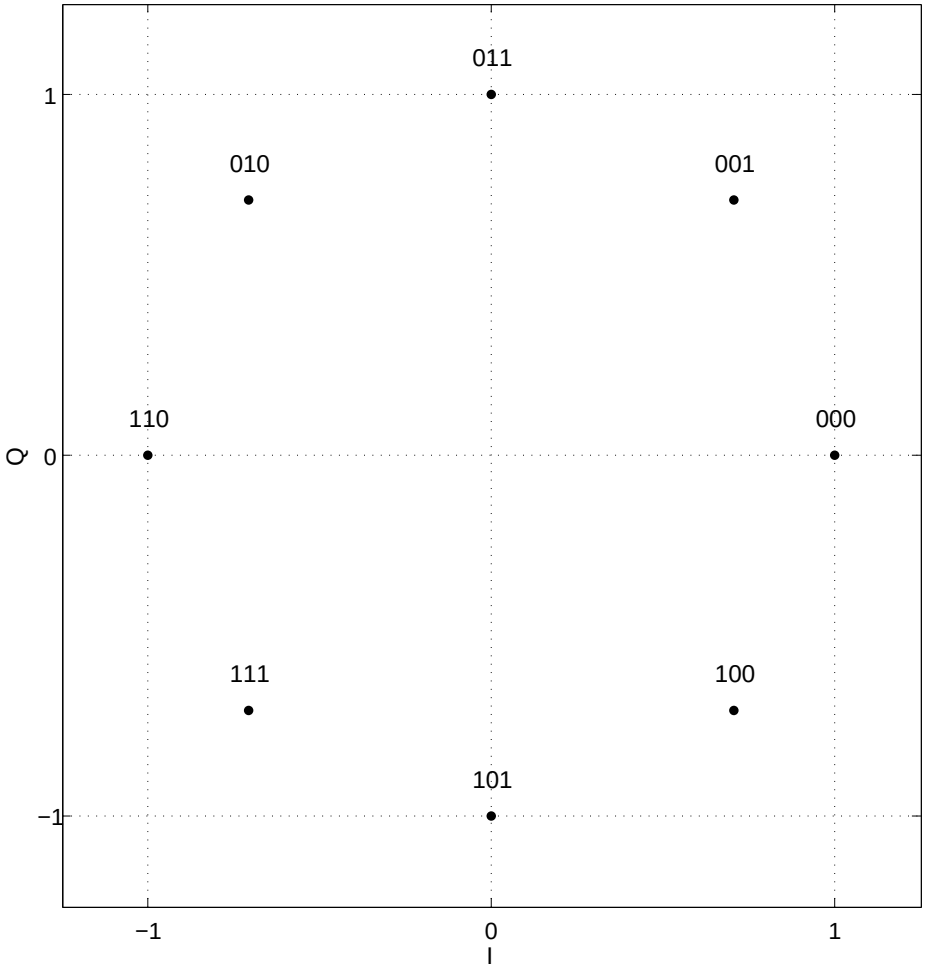


Figure 7: 8PSK constellation diagram.

be achieved by the Digital Quadrature Amplitude Modulation (digital-QAM) of the signal $v_8(t)$ onto an in-phase carrier and the signal $-v_9(t)$ onto a quadrature-phase carrier.

a) State the symbol rate R_s that is required.

b) Determine the duration of the resultant 8PSK signal.

c) Sketch a time-domain plot of the signal $v_8(t)$, remembering to annotate both axes.

d) Sketch a time-domain plot of the signal $v_9(t)$, remembering to annotate both axes.

e) State the bandwidth that is required for the resultant 8PSK signal, as well as the range of frequencies that it occupies.

f) Determine the bandwidth efficiency of the described 8PSK scheme.

Appendix

The following Matlab code can be used to generate the signals $v_1(t)$, $v_2(t)$ and $v_3(t)$ shown in Figures 1a. 1b and 3a, respectively.

```
% Seed Matlab's random number generator
randn('seed',1); % Gives v1(t)
%randn('seed',2); % Gives v2(t)
%randn('seed',3); % Gives v3(t)

% Choose a simulation frequency
f_s = 1000;

% Choose the maximum frequency present in our signal
f_max = 10;

% Determine the time instants to simulate
t = 0:1/f_s:0.3;

% Generate a white Gaussian noise signal
v = randn(size(t));

% If filtering is necessary...
if f_s > 2*f_max
        % Setup a low pass filter to bandlimit the noise
        B = fir1(100*ceil(f_s/f_max), 2*f_max/f_s);

        % Filter the noise signal
        v = filter2(B,v);

        % Set the signal's normalised average power to 1
        v = v*sqrt(length(t)/sum(v.^2));
end

% Plot the signal
plot(t,v)
```