QUESTION

Calculate the delta values for the followind two exact solutions of the Black-Scholes equation:

(a) $V(S,t) = AS$

(b) $V(S,t) = A\exp(rt)$

Comment on the associated trading strategies.

ANSWER

(a) $V(S,t) = AS$, A =const. (Asset only portfolio)

This solves Black-Scholes since:

$\frac{\partial v}{\partial t} = 0$, $\frac{\partial V}{\partial S} = A$, $\frac{\partial^2 V}{\partial S^2} = 0$

Therefore in Black-Scholes: $\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial v}{\partial S} - rV = 0$

LHS=$0 + \frac{1}{2}\sigma^2 S^2 \times 0 + rSA - rAS = 0$ =RHS

$\Delta = \frac{\partial V}{\partial S} = A \equiv$ amount of underlying asset at each point in time in portfolio: obvious from value V.

(b) $V(S,t) = Ae^{rt}$, A =const. (Risk free solution).

$\frac{\partial V}{\partial t} = Are^{rt}$, $\frac{\partial v}{\partial S} = 0$, $\frac{\partial^2 v}{\partial S^2} = 0$

Therefore in Black-scholes

LHS= $Are^{rt} + \frac{1}{2}\sigma^2 S^2 \times 0 + rS0 - rAe^{rt} = 0$ =RHS

$\Delta = \frac{\partial V}{\partial S} = 0 \rightarrow$ as risk free solution, no risky assets held.