QUESTION

Solve the following linear programming problem.

Maximize	$z = 18x_1 + 3x_2 + x_3 + 16x_4 + 3x_5$
subject to	$4x_1 + 2x_2 - x_3 + 3x_4 + x_5 \leq 24$
	$8x_1 + x_2 + x_3 + 4x_4 + 4x_5 \leq 30$
	$0 \leq x_1 \leq 3$
	$0 \leq x_2 \leq 9$
	$0 \leq x_3 \leq 10$
	$0 \leq x_4 \leq 6$
	$0 \leq x_5 \leq 15.$

(i) For the first constraint, give the range of values for the right hand side within which the optimal basis remains unaltered. Also, perform this ranging analysis for the upper bound constraint $x_4 \leq 6$.

(ii) If the last constraint is altered to become $-3 \leq x_5 \leq 20$, find how your solution is affected.

ANSWER

Add slack variables $s_1 \geq 0, s_2 \geq 0$, and use the bounded variable simplex method.

Basic	z	x_1	x_2	x_3	x_4	x_5	s_1	s_2	
s_1	0	4	2	-1	3	1	1	0	24
s_2	0	8	1	1	4	4	0	1	30
	1	-18	-3	-16	-3	0	0	0	

Substitute $x_1' = 3 - x_1$

Basic	z	x_1'	x_2	x_3	x_4	x_5	s_1	s_2	
s_1	0	-4	2	-1	3	1	1	0	12
s_1	0	8	1	1	4	4	0	1	6
	1	18	-3	0	13	0	4	78	

Basic	z	x_1'	x_2	x_3	x_4	x_5	s_1	s_2	
s_1	0	2	$\frac{5}{4}$	$-\frac{7}{4}$	0	-2	1	$-\frac{3}{4}$	$\frac{15}{2}$
x_4	0	-2	$\frac{1}{4}$	$\frac{1}{4}$	1	1	0	$\frac{1}{4}$	$\frac{3}{2}$
	1	-14	1	3	0	13	0	4	78

Substitute $x_4' = 6 - x_4$

Basic	z	x_1'	x_2	x_3	x_4'	x_5	s_1	s_2	
s_1	0	0	$\frac{3}{2}$	$-\frac{3}{2}$	-1	-1	1	$-\frac{1}{2}$	$9 - 6 = 3$
x_1'	0	1	$-\frac{1}{8}$	$-\frac{1}{8}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{8}$	$-\frac{3}{4} + 3 = \frac{9}{4}$
	1	0	$-\frac{3}{4}$	$\frac{5}{4}$	7	6	0	$\frac{9}{4}$	$\frac{135}{2} + 42 = \frac{219}{2}$

Basic	z	x_1	x_2	x_3	x_4	x_5	s_1	s_2	
x_2	0	0	1	-1	$-\frac{2}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	2	
x_1'	0	1	0	$-\frac{1}{4}$	$\frac{5}{12}$	$-\frac{7}{12}$	$\frac{1}{12}$	$-\frac{1}{6}$	$\frac{5}{2}$
	1	0	0	$\frac{1}{2}$	$\frac{13}{2}$	$\frac{11}{2}$	$\frac{1}{2}$	2	111

The optimal solution is

$$x_1' = \frac{5}{2},\ x_2 = 2,\ x_3 = 0,\ x_4' = 0,\ x_5 = 0$$

$$x_1 = \frac{1}{2},\ x_2 = 2,\ x_3 = 0,\ x_4 = 6,\ x_5 = 0,\ z = 111$$

(i) If the first RHS becomes $24 + \delta$, the RJHS columns in the final tableau becomes

$$\begin{array}{rcl} 2 & + & \frac{2}{3}\delta \\ \frac{5}{2} & + & \frac{1}{12}\delta \\ 11 & + & \frac{1}{2}\delta \end{array}$$

For non-negativity $\delta \geq -3,\ \delta \geq -30$

For

$$x_1' \leq 3 : \frac{5}{2} + \frac{1}{12}\delta \leq 3\ \delta \leq 6$$

$$x_2 \leq 9 : 2 + \frac{2}{3}\delta \leq 9\ \delta \leq \frac{21}{2}$$

The required range is $-3 \leq \delta \leq 6$

If the current x_4' column is changed from $6 - x_4$ to $6 + \delta - x_4$, the RHS's become

$$\begin{array}{rcl} 2 & - & \frac{2}{3}\delta \\ \frac{5}{2} & + & \frac{5}{12}\delta \\ 11 & + & \frac{13}{2}\delta \end{array}$$

For non-negativity $\delta \leq 3,\ \delta \geq -6$

For

$$x_1' \leq 3 \; \frac{5}{2} + \frac{5}{12}\delta \leq 3 \; \delta \leq \frac{6}{5}$$

$$x_2 \leq 9 \; 2 - \frac{2}{3}\delta \leq 9 \; \delta \geq -\frac{21}{2}$$

The required range is $-6 \leq \delta \leq \frac{6}{5}$

(ii) Change the variable: $\overline{x}_5 = x_5 + 3$, so that $0 \leq \overline{x}_5 \leq 23$. The tableau is

$\overline{x}_5 - 3$	
$-\frac{2}{3}$	2
$-\frac{7}{12}$	$\frac{5}{2}$
$\frac{11}{2}$	111

which becomes

$\overline{x}_5$	
$-\frac{2}{3}$	0
$-\frac{7}{12}$	$\frac{3}{4}$
$\frac{11}{2}$	$127\frac{1}{2}$

Thus z increases by $\frac{33}{2}$, and the values of x_1 and x_2 become $x_1 = \frac{9}{4},\ x_2 = 0$.