

**Question**

Given  $\vec{OA} = (1, 0, -1)$ ,  $\vec{OB} = (1, 2, 3)$  and  $\vec{OC} = (0, 1, 2)$ . Find

- (a) the direction vector through  $A$  and  $B$ ;
- (b) the vector equation of the line  $AB$ ;
- (c) the Cartesian equation of the line  $AB$ .

**Answer**

$$\vec{OA} = (1, 0, -1), \quad \vec{OB} = (1, 2, 3), \quad \vec{OC} = (0, 1, 2)$$

(a)  $\vec{AB} = \vec{OB} - \vec{OA} = (1, 2, 3) - (1, 0, -1) = (0, 2, 4) = \mathbf{m}$

(b) vector equation of the line  $AB$  is

$$\begin{aligned} \mathbf{r} &= (1, 0, -1) + \lambda(0, 2, 4) \\ &= (1, 2\lambda, -1 + 4\lambda) \\ &= \mathbf{r}_0 + \lambda\mathbf{m} \end{aligned}$$

(c) Cartesian equation is

$$\begin{array}{rcl} x - x_0 & = & \lambda m_x & & x & = & 1 \\ y - y_0 & = & \lambda m_y & \Rightarrow & y - y_0 & = & 2\lambda = y \\ z - z_0 & = & \lambda m_z & & z - z_0 & = & 4\lambda = z + 1 \end{array}$$

Hence  $(z + 1) = 2(2\lambda) = 2y$

So  $x = 1, z - 2y + 1 = 0$  are the equations of the line.