Question
Consider the vector $\mathbf{r}(t) = a\cos\omega t\mathbf{i} + a\sin\omega t\mathbf{j} + a\omega t\mathbf{k}$
Find: (a) the velocity vector; (b) the acceleration vector; (c) the speed.

Answer

$$\mathbf{r}(t) = a\cos\omega t\mathbf{i} + a\sin\omega t\mathbf{j} + a\omega t\mathbf{k}$$

[This is helical motion]

(a) $\mathbf{v}(t) = \frac{d\mathbf{r}}{dt} = (-\omega a\sin\omega t, a\omega\cos\omega t, a\omega)$

(b) $\mathbf{a}(t) = \frac{d^2\mathbf{r}}{dt^2} = (-\omega^2 a^2\cos\omega t, -a\omega^2\sin\omega t, 0)$

(c)

$$\begin{aligned} |\mathbf{v}(t)| = \text{speed} &= [\omega^2 a^2\sin^2\omega t + \omega^2 a^2\cos^2\omega t + \omega^2 a^2]^{\frac{1}{2}} \\ &= \omega a[1+1]^{\frac{1}{2}} \\ &= \sqrt{2}\omega a \end{aligned}$$