Question

Find the volume of the parallelipiped with sides given by the vectors $\mathbf{a} = (1, 1, -1)$ $\mathbf{b} = (1, -1, 1)$, $\mathbf{c} = (-1, 1, 1)$

Answer

$$\mathbf{a} = (1, 1, -1) \quad \mathbf{b} = (1, -1, 1) \quad , \mathbf{c} = (-1, 1, 1)$$

$\text{vol} = |(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}|$

$\mathbf{a} \times \mathbf{b} = ((1 \times 1) - (-1 \times -1), (-1 \times 1) - (1 \times 1), (1 \times -1) - (1 \times 1)) = (0, -2, -2)$

$\mathbf{a} \times \mathbf{a} \cdot \mathbf{a} = (0, -2, -2) \cdot (-1, 1, 1) = -4$

Hence the volume is equal to 4.