

Partial Differentiation
Limits

Question

Evaluate the given limit. If the limit does not exist, explain why.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x}{x^2 + y^2}$$

Answer

If $f(x, y) = \frac{x}{x^2 + y^2}$

Then $|f(x, 0)| = |1/x| \rightarrow \infty$ as $x \rightarrow 0$.

But $|f(0, y)| = 0 \rightarrow 0$ as $y \rightarrow 0$.

And so $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ does not exist.