

**Partial Differentiation**  
***Limits***

**Question**

If  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^m y^n}{(x^2 + y^2)^p}$  is to exist, what condition must the integers  $m$ ,  $n$  and  $p$  satisfy, given that  $m, n, p > 0$ .

Prove your answer.

**Answer**

As

$$|x| \leq \sqrt{x^2 + y^2}$$
$$\text{and } |y| \leq \sqrt{x^2 + y^2}$$

This gives

$$\left| \frac{x^m y^n}{(x^2 + y^2)^p} \right| \leq \frac{(x^2 + y^2)^{(m+n)/2}}{(x^2 + y^2)^p}$$
$$= (x^2 + y^2)^{-p+(m+n)/2}$$

And  $(x^2 + y^2)^{-p+(m+n)/2} \rightarrow 0$  as  $(x, y) \rightarrow (0, 0)$   
provided  $m + n > 2p$

$$\Rightarrow \lim_{(x,y) \rightarrow (0,0)} \frac{x^m y^n}{(x^2 + y^2)^p} = 0.$$