Partial Differentiation
Limits

Question

Explain how the function

$$f(x,y) = \frac{x^3 - y^3}{x - y}, \quad (x \neq y)$$

can be defined along the line $y = x$, so that it becomes continuous at all points of the xy-plane.

Answer

$$f(x,y) = \frac{x^3 - y^3}{x - y} = x^2 + xy + y^2$$

if $x \neq y$.

But $x^2 + xy + y^2 = 3x^2$ on the line $y = x$.

Therefore define $f(x,x) = 3x^2$ and the function will equal $x^2 + xy + y^2$ at all points. It will therefore be continuous at all points.

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