QUESTION
Define the following terms:

(i) subgroup

(ii) left coset

(iii) the order of an element g in a group G.

State Lagrange's theorem and use it to show that if H and K are finite subgroups of a group G with $|H|$ and $|K|$ coprime, then $H \cap K = \{e\}$.
Show that, if in addition H and K are both normal subgroups of G. then for any elements $h \in H$ and $k \in K$, the product $h^{-1}k^{-1}hk = e$.
Give an example of a finite group G which contains a non-normal subgroup H.
ANSWER

(i) Let $(G, e, *)$ be a group. A subgroup of $(G, e, *)$ is a subset $H < F$ such that

  (s1) $e \in H$

  (s2) If $h_1, h_2 \in H$ then $h_1 8 h_2 \in H$

  (s3) If $h \in H$ then $h^{-1} \in H$.

(ii) if H is a subgroup of G and $g \in G$ then the left coset gH is the subset $\{gh | h \in H\}$

(iii) The order of g is the least prime integer n such that $g^n = e$, or is ∞ if there is no such n.

Lagrange's Theorem
Let G be a finite group and $H < G$ then $|H|$ divides $|G|$.
$H \cap K$ is a subgroup of H so $|H \cap K|$ divides $|H|$. Similarly it divides $|K|$.
Since their greatest common divisor is 1, $|H \cap K| = 1$.
The element $h^{-1}k^{-1}h \in K$ since $K \triangle G$ so $h^{-1}k^{-1}hk \in K$. Similarly $h^{-1}k^{-1}hk \in H$ so $h^{-1}k^{-1}hk \in H \cap K = \{e\}$.
Let $G = D_n,\ H = \langle \sigma_i \rangle\ \ \rho^{-1}H\rho = \langle \sigma_{i+1} \rangle = \langle \sigma_i \rangle$ or similar. ρ=rotation and σ_i=reflection.