Question

A particle moves under the influence of the force $F = -kx + \dfrac{kx^3}{\alpha^2}$, where k and α are constants and $k > 0$.

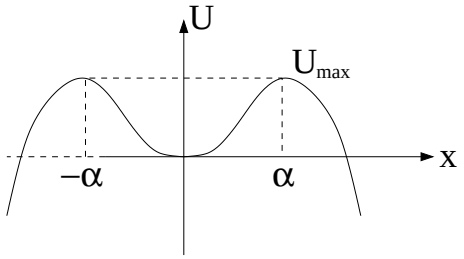
(a) Determine the potential $U(x)$ and discuss the motion.

(b) What happens if the total energy $E = \dfrac{k\alpha^2}{4}$?

Answer

(a) $U = -\int F\,dx = -\int \left(-kx + \dfrac{kx^3}{\alpha^2}\right) dx$

$$U(x) = \frac{1}{2}kx^2 - \frac{kx^4}{4\alpha^2}$$



If the particle has total energy less than $U_{\max}$ it will either oscillate about the origin or go off to $x = \pm\infty$ depending on the initial position.

Otherwise, if $U > U_{\max}$, it will move off to $\pm\infty$.

If $U = U_{\max}$ see (b) below.

(b) If $U = U_{\max}$. Either the motion is in $-\alpha < x < \alpha$, in which case after infinite time the particle reaches $x = \alpha$ or $x = -\alpha$; or if $|x| > \alpha$ the particle moves off to $x = +\infty$ or $-\infty$ with increasing speed.