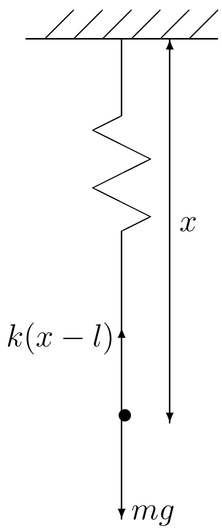
Question

A particle of mass m hangs vertically on the end of a spring of stiffness k and natural length l. The particle is displaced vertically downwards a distance $\frac{l}{2}$ and released from rest.

(a) What is the maximum height achieved by the particle in the subsequence motion?

(b) Show that the particle oscillates up and down with a frequency $\sqrt{\dfrac{k}{m}}$.

Answer



Using Newton's 2nd law:

$$\begin{aligned} m\ddot{x} &= mg - k(x-l) \\ \ddot{x} &= g + \frac{kl}{m} - kmx \quad (*) \\ \text{Energy : K.E.} + \text{P.E.} &= \text{constant} \\ \frac{1}{2}m\dot{x}^2 + \frac{1}{2}k(x-l)^2 &= 0 + \frac{1}{2}k\left(\frac{3}{2}l - l\right) \end{aligned}$$

Since initially $\dot{x} = 0$ and $x = \frac{3}{2}l$

(a) At the maximum height $v = 0$.

Therefore $\dfrac{1}{2}k(x-l)^2 = \dfrac{1}{2}k\left(\dfrac{l}{2}\right)^2 \Rightarrow x = \dfrac{l}{2}$

(b) Solving (*) gives: $x = l + \frac{mg}{k} + A\cos\left(\sqrt{\frac{k}{m}}t + B\right)$, where A and B are constants and the frequency is $\sqrt{\frac{k}{m}}$