

Question

A rocket starts from rest in free space by emitting mass. At what fraction of the initial mass is the momentum of the rocket a maximum?

Answer

From theory $v = v_0 + u \ln \left(\frac{m_0}{m} \right)$

The rocket starts from rest so $v_0 = 0$

Rocket momentum $p = mv = mu \ln \left(\frac{m_0}{m} \right)$

Where is p a maximum?

$$\frac{dp}{dm} = u \left[\ln \left(\frac{m_0}{m} \right) - 1 \right]$$

Therefore p is max/min when $m = e^{-1}m_0$

Check that $\frac{d^2p}{dm^2} < 0$ to confirm that p is a maximum when $m = e^{-1}m_0$