Applications of Partial Differentiation
Extremes

Question

Find the minimum and maximum values of

$$f(x, y) = \frac{x}{(1 + x^2 + y^2}$$

Answer

$$
\begin{aligned}
f(x, y) &= \frac{x}{(1 + x^2 + y^2)} \\
f_1(x, y) &= \frac{1 + y^2 - x^2}{(1 + x^2 + y^2)^2} \\
f_2(x, y) &= \frac{-2xy}{(1 + x^2 + y^2)^2} \\
&\text{i.e. y } = 0 \text{ or z } = 0 \text{ or x } = \pm\frac{1}{\sqrt{2}}
\end{aligned}
$$

For critical points, $x^2 - y^2 = 1$ and $xy = 0$. The critical points are $(\pm 1, 0)$.
$f(\pm 1, 0) = \pm\frac{1}{2}$.
Since $f(x, y) \to 0$ as $x^2 + y^2 \to \infty$, the maximum and minimum values of f
are $1/2$ and $-1/2$ respectively.