

Applications of Partial Differentiation
Extremes

Question

Find and classify the critical points of the function

$$f(x, y) = \cos(x + y)$$

Answer

$$f(x, y) = \cos(x + y)$$

$$f_1 = -\sin(x + y) = f_2$$

All points on the lines $x + y = n\pi$ (n is an integer) are critical points.

If n is even: $f = 1$ at such points.

If n is odd: $f = -1$ there.

Since $-1 \leq f(x, y) \leq 1$ at all points in \mathbf{R}^2 , f must have local and absolute maximum values at points $x + y = n\pi$ with n even, and local and absolute minimum values at such points with n odd.