

Applications of Partial Differentiation
Extremes

Question

Find and classify the critical points of the function

$$f(x, y) = x^4 + y^4 - 4xy$$

Answer

$$\begin{aligned} f_1 &= 4(x^3 - y) \\ f_2 &= 4(y^3 - x) \\ A &= f_{11} = 12x^2 \\ B &= f_{12} = -4 \\ C &= f_{22} = 12y^2. \end{aligned}$$

For critical points: $x^3 = y$ and $y^3 = x$. Thus $x^9 = x$, or $x(x^8 - 1) = 0$, and $x = 0, 1$, or -1 .

The critical points are $(0, 0)$, $(1, 1)$ and $(-1, -1)$.

At $(0, 0)$: $B^2 - AC = 16 - 0 > 0$, so $(0, 0)$ is a saddle point.

At $(1, 1)$ and $(-1, -1)$: $B^2 - AC = 16 - 144 < 0$, $A > 0$ so f has local maxima at these points.