Applications of Partial Differentiation
Extremes

Question

Find and classify the critical points of the function

$$f(x, y) = \frac{1}{1 - x + y + x^2 + y^2}$$

Answer

$$
\begin{aligned}
f(x, y) &= \frac{1}{1 - x + y + x^2 + y^2} \\
&= \frac{1}{\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 + \frac{1}{2}}
\end{aligned}
$$

Obviously f has maximum value 2 at $\left(\frac{1}{2}, -\frac{1}{2}\right)$.

Since

$$
\begin{aligned}
f_1(x, y) &= \frac{1 - 2x}{(1 - x + y + x^2 + y^2)^2} \\
f_2(x, y) &= -\frac{1 + 2y}{(a - x + y + x^2 + y^2)^2}
\end{aligned}
$$

$\left(\dfrac{1}{2}, -\dfrac{1}{2}\right)$ is the only critical point of f.