Applications of Partial Differentiation
Extremes

Question
Find and classify the critical points of the function

$$f(x, y) = x^2 + 2y^2 - 4x + 4y$$

Answer

$$\begin{aligned} f_1(x, y) &= 2x - 4 = 0 \quad \text{if x} = 2 \\ f_2(x, y) &= 4y + 4 = 0 \quad \text{if y} = -1 \end{aligned}$$

Critical point is $(2, -1)$
Since $f(x, y) \to \infty$ as $x^2 + y^2 \to \infty$, f has a local (and absolute) minimum value at that critical point.