Question

The following question is adapted from R.A.Adams "Calculus" exercises 19.2

Solve the following differential equations

1)	$\dfrac{dy}{dx} = \dfrac{y}{2x}$	2)	$\dfrac{dy}{dx} = \dfrac{3y-1}{x}$ $\quad (*)$
3)	$\dfrac{dy}{dx} = \dfrac{x^2}{y^2}$	4)	$\dfrac{dy}{dx} = x^2y^2$
5)	$\dfrac{dy}{dx} = \dfrac{x^2}{y^3}$	6)	$\dfrac{dy}{dx} = x^3y^4$ $\quad (*)$
7)	$\dfrac{dY}{dt} = tY$	8)	$\dfrac{dx}{dt} = e^x \cos t$
9)	$\dfrac{dy}{dx} = 1 - y^2$	10)	$\dfrac{dy}{dx} = 1 + y^2$
11)	$\dfrac{dy}{dt} = 2 + e^y$	12)	$\dfrac{dy}{dx} = y^2(1-y)$ $\quad (*)$
13)	$\dfrac{dy}{dx} = \sin x \cos^2 y$	14)	$x\dfrac{dy}{dx} = y \ln x$

Answer

(odd solutions in Adams "Calculus")

(1-14 are separable)

2) $$\frac{dy}{dx} = \frac{3y-1}{x} \Rightarrow \int \frac{dy}{3y-1} = \int \frac{dx}{x} \Rightarrow \frac{1}{3}\ln(3y-1) = \ln x + c$$

$$(3y-1)^{\frac{1}{3}} = e^c x \Rightarrow 3y - 1 = kx^3 \Rightarrow y = Ax^3 + \frac{1}{3}$$

4) $$\frac{dy}{dx} = x^2y^2 \Rightarrow \int \frac{dy}{y^2} = \int x^2 dx \Rightarrow -\frac{1}{y} = \frac{1}{3}x^3 + c$$

$$y = \frac{1}{A - \frac{1}{3}x^3}$$

6) $$\frac{dy}{dx} = x^3y^4 \Rightarrow \int \frac{dy}{y^4} = \int x^3 dx \Rightarrow -\frac{1}{3y^3} = \frac{1}{4}x^4 + c$$

$$y^3 = \frac{1}{A - \frac{3}{4}x^4} \Rightarrow y = \left(\frac{1}{A - \frac{3}{4}x^4}\right)^{\frac{1}{3}}$$

8) $\frac{dx}{dt} = e^x \cos t \Rightarrow \int e^{-x} dx = \int \cos t dt \Rightarrow -e^{-x} = \sin t + c$

$e^{-x} = -\sin t - c \Rightarrow x = -\ln(-\sin t - c)$

10) $\frac{dy}{dx} = 1 + y^2 \Rightarrow \int \frac{dy}{1+y^2} = \int dx \Rightarrow \tan^{-1} y = x + c$

$y = \tan(x + c)$

12) $\frac{dy}{dx} = y^2(1-y) \Rightarrow \int \frac{dy}{y^2(1-y)} = \int dx$ then partial fractions

$\int \frac{1}{y^2} + \frac{1}{y} + \frac{1}{1-y} dy = x + c \Rightarrow -\frac{1}{y} + \ln y - \ln(1-y) = x + 1$

can't simplify.

14) $x\frac{dy}{dx} = y \ln x \Rightarrow \int \frac{1}{y} dy = \int \frac{\ln x}{x} dx$ (use substitution $\ln x = q$ to do integration)

$\ln y = \frac{1}{2}(\ln x)^2 + c \Rightarrow y = e^c e^{\frac{1}{2}(\ln x)^2)}$