Question

Find the general solutions of the following differential equations:

1. $\dfrac{dy}{dx} = \dfrac{y-1}{x+3} \qquad (*)$
2. $\dfrac{dy}{dx} = \dfrac{1}{(x-1)y^3}$
3. $\dfrac{dy}{dx} = 3x^2(1+y^2)$

Answer

a) $\dfrac{dy}{dx} = \dfrac{y-1}{x+3} \Rightarrow \displaystyle\int \frac{dy}{y-1} = \int \frac{dx}{x+3}$

$$\begin{aligned} \Rightarrow \ln(y-1) &= \ln(x+3) + c \\ y-1 &= e^c(x+3) \\ y-1 &= k(x+3) \\ y &= k(x+3)+1 \end{aligned}$$

For any constant k.

b) $\dfrac{dy}{dx} = 1(x-1)y^3 \Rightarrow \displaystyle\int y^3 dy = \int \frac{dx}{x-1} \Rightarrow \frac{1}{4}y^4 = \ln(x-1) + c$

$$\begin{aligned} \Rightarrow y &= \pm(4\ln(x-1)+c)^{\frac{1}{4}} \\ y &= \pm\sqrt{2}(\ln(k(x-1)))^{\frac{1}{4}} \end{aligned}$$

For any constant k.

c) $\dfrac{dy}{dx} = 3x^2(1+y^2) \Rightarrow \displaystyle\int \frac{dy}{1+y^2} = \int 3x^2 dx \Rightarrow \tan^{-1} y = x^3 + c$

$y = \tan(x^3 + c)$