

Question

Find the inverses of the following matrices and verify that they are correct.

(i) $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(ii) $B = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

(iii) $C = \begin{pmatrix} 2 & -2 \\ 6 & -3 \end{pmatrix}$

Answer

(i) $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow \det A = 0 - 1 = -1$

and $A^{-1} = \frac{1}{-1} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

i.e., $A^{-1} = A!!!$

Check $A^{-1}A = I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$A^{-1}A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$

Should also check $AA^{-1} = I_2$ (obvious here)

(ii) $B = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow \det B = 1 \times 0 - 1 \times 1 = -1$

and $B^{-1} = \frac{1}{-1} \begin{pmatrix} 0 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$

Check $B^{-1}B = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2 \checkmark$

$BB^{-1} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2 \checkmark$

$$\text{(iii)} \quad C = \begin{pmatrix} 2 & -2 \\ 6 & -3 \end{pmatrix} \Rightarrow \det C = -6 - (-12) = 6$$

$$\text{and } C^{-1} = \frac{1}{6} \begin{pmatrix} -3 & 2 \\ -6 & 2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{3} \\ -1 & \frac{1}{3} \end{pmatrix}$$

$$\text{Check } C^{-1}C = \begin{pmatrix} -\frac{1}{2} & \frac{1}{3} \\ -1 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 2 & -2 \\ 6 & -3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2 \quad \checkmark$$

$$CC^{-1} = \begin{pmatrix} 2 & -2 \\ 6 & -3 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} & \frac{1}{3} \\ -1 & \frac{1}{3} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2 \quad \checkmark$$