**Question**

Let $n \geq 4$ be an integer that is not prime. Show that the integers modulo $n$, $\mathbf{Z}_n$, is not a field.

**Answer**

Write $n$ as a product $n = a \cdot b$, where $2 \leq a, b < n$, so that $a$ and $b$ are not equal in $\mathbf{Z}_n$. Then, in $\mathbf{Z}_n$, the product $a \cdot b$ is 0, being a multiple of $n$. However, if $\mathbf{Z}_n$ were a field, then $a$ would have a multiplicative inverse $a^{-1}$, and we could multiply both sides of $a \cdot b = 0$ on the left to obtain $a^{-1} \cdot a \cdot b = a^{-1} \cdot 0$, which simplifies to $b = 0$. This contradicts the choice of $b$ to satisfy $2 \leq b < n$, and so $a$ has no multiplicative inverse, contradicting the definition of a field.