

Question

Show that if f is continuous then $\{x|f(x) \leq c\}$ is a closed set. Deduce that f is measurable.

Answer

Let a be a point of accumulation of $A = \{x|f(x) \leq c\}$. Then there exists $a_i \in A$, $a_i \rightarrow a$, $f(a_i) \leq c$ and f is continuous, so $f(a_i) \rightarrow f(a)$ therefore $f(a) \leq c$. i.e. $a \in A$.

Therefore A is closed and so measurable, thus f is measurable.