Vector Calculus
Grad, Div and Curl Identities

Question

Verify that $\underline{F} \times \underline{G}$ is solenoidal for smooth and conservative vector fields $\underline{F}$ and $\underline{G}$. Also find a vector potential for $\underline{F} \times \underline{G}$.

Answer

$$
\begin{aligned}
\underline{F} &= \nabla\phi \\
\text{and } \underline{G} &= \nabla\psi \\
\Rightarrow \nabla \times \underline{F} &= \underline{0} \\
\text{and } \nabla \times \underline{G} &= \underline{0} \\
\Rightarrow \nabla \bullet (\underline{F} \times \underline{G}) &= (\nabla \times \underline{F}) \bullet \underline{G} + \underline{F} \bullet (\nabla \times \underline{G}) \\
&= 0
\end{aligned}
$$

and so $\underline{F} \times \underline{G}$ is solenoidal.

It can be seen that

$$\nabla \times (\phi\nabla\psi) = \nabla\phi \times \nabla\psi = \underline{F} \times \underline{G}.$$

so $\phi\nabla\psi$ is a vector potential for $\underline{F} \times \underline{G}$, as is $-\psi\nabla\phi$.