Vector Calculus
Grad, Div and Curl Identities

Question

Given that ϕ and ψ are both harmonic functions, show that

$$\phi\nabla\psi - \psi\nabla\phi$$

is solenoidal.

Answer

Given that $\nabla^2\phi = 0$ and $\nabla^2\psi =)$

$$
\begin{aligned}
\Rightarrow \nabla \bullet (\phi\nabla\psi - \psi\nabla\phi) \;=\;& \nabla\phi \bullet \nabla\psi + \phi\nabla^2\psi \\
& -\nabla\psi \bullet \nabla\phi - \psi\nabla^2\phi = 0
\end{aligned}
$$

Therefore $\phi\nabla\psi - \psi\nabla\phi$ is solenoidal.