Vector Calculus
Grad, Div and Curl Identities

Question
It is given that $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$ and that $\underline{c}$ is a constant vector.
Show that

$$\begin{aligned} \nabla \bullet (\underline{c} \times \underline{r}) &= 0 \\ \nabla \times (\underline{c} \times \underline{r}) &= 2\underline{c} \\ \nabla(\underline{c} \bullet \underline{r}) &= \underline{c} \end{aligned}$$

Answer

$$\begin{aligned} \nabla \bullet \underline{r} &= 3 \\ \nabla \times \underline{r} &= \underline{0} \end{aligned}$$
$$\nabla r = \frac{\underline{r}}{r}$$

$\underline{c}$ is a constant vector, hence its div and curl both equal zero.

$$\begin{aligned} \Rightarrow \nabla \bullet (\underline{c} \times \underline{r}) &= (\nabla \times \underline{c}) \bullet \underline{r} - \underline{c} \bullet (\nabla \times \underline{r}) = \underline{0} \\ \nabla \times (\underline{c} \times \underline{r}) &= (\nabla \bullet \underline{r})\underline{c} + (\underline{r} \bullet \nabla)\underline{c} - (\nabla \bullet \underline{c})\underline{r} - (\underline{c} \bullet \nabla)\underline{c} \\ &= 3\underline{c} + \underline{0} - \underline{0} - \underline{c} = 2\underline{c} \end{aligned}$$

$$\begin{aligned} \nabla(\underline{c} \bullet \underline{r}) &= \underline{c} \times (\bullet \times \underline{r}) + \underline{r} \times (\nabla \times \underline{c}) + (\underline{c} \bullet \nabla)\underline{r} + (\underline{r} \bullet \nabla)\underline{c} \\ &= \underline{0} + \underline{0} + \underline{c} + \underline{0} = \underline{c} \end{aligned}$$