Vector Calculus
Grad, Div and Curl Identities

Question
If the field lines of the vector field $\underline{F}(x, y, z)$ are parallel straight lines, what does this tell you about div$\underline{F}$ and curl$\underline{F}$?
Answer
If the field lines are parallel straight lines, in the direction of the non-zero vector $\underline{a}$, where $\underline{a}$ is a constant, then

$$\underline{F}(x, y, z) = \phi(x, y, z)\underline{a}$$

with ϕ being a smooth scalar field. It is also given that

$$\begin{aligned} \text{div}\underline{F} &= \text{div}(\phi\underline{a}) = \nabla\phi \bullet \underline{a} \\ \text{curl}\underline{F} &= \text{curl}(\phi\underline{a}) = \nabla\phi \times \underline{a}. \end{aligned}$$

As $\nabla\phi$ is an arbitrary gradient, so div$\underline{F}$ can take any value. However, curl$\underline{F}$ will be perpendicular to $\underline{a}$, and so also perpendicular to $\underline{F}$.