

**Vector Calculus**  
*Grad, Div and Curl Identities*

**Question**

If  $f$ ,  $g$  and  $h$  are smooth vector fields, show that

$$\nabla \bullet (f(\nabla g \times \nabla h)) = \nabla f \bullet (\nabla g \times \nabla h).$$

**Answer**

$$\begin{aligned} \nabla \bullet (f(\nabla g \times \nabla h)) &= \nabla f \bullet (\nabla g \times \nabla h) \\ &\quad + f \nabla \bullet (\nabla g \times \nabla h) \\ &= \nabla f \bullet (\nabla g \times \nabla h) + f((\nabla \times \nabla g) \bullet \nabla h \\ &\quad - \nabla g \bullet (\nabla \times \nabla h)) \\ &= \nabla f \bullet (\nabla g \times \nabla h) + \underline{0} - \underline{0} \\ &= \nabla f \bullet (\nabla g \times \nabla h). \end{aligned}$$