Vector Calculus

Grad, Div and Curl Identities

Question

The smooth vector field $\underline{F}$ is irrotational and solenoidal on $\Re^3$. Show that both the three components of $\underline{F}$ and the scalar potential for $\underline{F}$ are harmonic functions in $\Re^3$.

Answer

div$\underline{F} = 0$ and curl$\underline{F} = \underline{0}$, $\Rightarrow \nabla^2 \underline{F} = 0$. So the components of $\underline{F}$ are harmonic functions.

If $\underline{F} = nabla\phi$

$$\Rightarrow \nabla^2 \phi = \nabla \bullet \nabla \phi = \nabla \bullet \underline{F} = 0$$

therefore ϕ is also harmonic.