<div align="center">

Vector Calculus
Grad, Div and Curl Identities

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Question

It is given that ϕ and ψ are scalar fields and $\underline{F}$ and $\underline{G}$ are vector fields. They are all assumed to be smooth functions. Prove the following identity

$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$$

Answer

$$
\begin{aligned}
\nabla(\phi\psi) &= \frac{\partial}{\partial x}(\phi\psi) + \frac{\partial}{\partial y}(\phi\psi) + \frac{\partial}{\partial z}(\phi\psi) \\
&= \left(\phi\frac{\partial\psi}{\partial x} + \frac{\partial\phi}{\partial x}\psi\right)\underline{i} + \cdots + \left(\phi\frac{\partial\psi}{\partial z} + \frac{\partial\phi}{\partial z}\psi\right)\underline{k} \\
&= \phi\nabla\psi + \psi\nabla\phi
\end{aligned}
$$