

QUESTION

(a) Find the following indefinite integrals

$$(i) \int \frac{1}{x+3} dx, \quad (ii) \int \frac{x+1}{x^2+2x+2} dx, \quad (iii) \int \frac{1}{x^2+2x+2} dx.$$

(b) Using partial fractions and the results in part (a) of this question, if appropriate, find

$$\int \frac{x^2 - 2x - 5}{(x+3)(x^2+2x+2)} dx.$$

ANSWER

(a) (i)

$$\int \frac{dx}{x+3} = \ln|x+3| + c$$

(ii)

$$\begin{aligned} \int \frac{x+1}{x^2+2x+2} dx &= \int \frac{\frac{1}{2}(2x+2)}{x^2+2x+2} dx \\ &= \frac{1}{2} \ln|x^2+2x+2| + c \end{aligned}$$

(iii)

$$\begin{aligned} \int \frac{dx}{x^2+2x+2} &= \int \frac{dx}{(x+1)^2+1^2} \\ &= \tan^{-1}(x+1) + c \end{aligned}$$

(b)

$$\begin{aligned} \frac{x^2 - 2x - 5}{(x+3)(x^2+2x+2)} &= \frac{A}{x+3} + \frac{Bx+C}{x^2+2x+2} \\ &= \frac{A(x^2+2x+2) + (Bx+C)(x+3)}{(x+3)(x^2+2x+2)} \end{aligned}$$

$$\begin{aligned} x^2 - 2x - 5 &= A(x^2+2x+2) + (Bx+C)(x+3) \\ x = -3; 9 + 6 - 5 &= A(9 - 6 + 2) + 0, \quad 10 = 5A, \quad A = 2 \\ x^2; 1 &= A + B, \quad B = 1 - A = -1 \\ \text{const.}; -5 &= 2A + 3C, \quad 3C = -5 - 2A = -9, \quad C = -3 \end{aligned}$$

Thus

$$\begin{aligned} & \int \frac{x^2 - 2x - 5}{(x + 3)(x^2 + 2x + 2)} dx \\ &= \int \left( \frac{2}{x + 3} - \frac{x + 3}{x^2 + 2x + 2} \right) dx \\ &= \int \frac{2}{x + 3} dx - \int \frac{x + 1}{x^2 + 2x + 2} dx - \int \frac{2}{x^2 + 2x + 2} dx \\ &= 2 \ln |x + 3| - \frac{1}{2} \ln |x^2 + 2x + 2| - 2 \tan^{-1}(x + 1) + c \end{aligned}$$