Question
A right-handed rectangular co-ordinate system is rotated through an angle of 120° about the line $x = y = z$, Find the matrix A of the transformation and show that $\det A = +1$. What is A^3?

Answer
If we perform this rotation successively 3 times we get back to where we started from. So $A^3 = I$.
The rotation permutes the axis $x_1x_2x_3 \to x_3x_1x_2$ So

$$\begin{pmatrix} x_3 \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$\det A = 1 \quad A^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

If the rotation is the other way $x_1x_2x_3 \to x_2x_3x_1$ then $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$