

Question

Let

$$A = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} \\ -1 & 0 & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{6}} \end{pmatrix}.$$

Verify that A is orthogonal. Suppose co-ordinates are related by $\mathbf{x} = A\mathbf{X}$. Find the \mathbf{X} equations of the images of the x_1 , x_2 and x_3 axes and verify that the images are mutually orthogonal.

AnswerVerify A orthogonal $A^T A = I$

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

So the images are:-

$$\left. \begin{array}{l} x_1 \text{ axis } \left(\frac{1}{\sqrt{3}}x_1, \frac{1}{\sqrt{2}}x_1, -\frac{1}{\sqrt{6}}x_1 \right) \\ x_2 \text{ axis } \left(\frac{1}{\sqrt{3}}x_2, -\frac{1}{\sqrt{2}}x_2, -\frac{1}{\sqrt{6}}x_1 \right) \\ x_3 \text{ axis } \left(-\frac{1}{\sqrt{3}}x_3, 0, -\frac{2}{\sqrt{6}}x_3 \right) \end{array} \right\} \text{Mutually orthogonal.}$$