

Question

Prove that if a matrix A and its inverse both have all their elements integers, then $\det A = \pm 1$

Answer

If A has all its entries integers then $\det A$ is an integer.

Ditto for $\det A^{-1}$ $1 = \det A \cdot \det A^{-1} = \det A \det A^{-1}$

therefore $\det A = \det A^{-1} = \pm 1$