Question
Let

$$A = \begin{pmatrix} -2 & 3 \\ 4 & -5 \end{pmatrix} \quad b = \begin{pmatrix} 1 & 0 \\ 3 & -2 \end{pmatrix}.$$

Evaluate AB, $BA, a, A^2 - B^2$, $(A - B)(A + B)$, $(A + B)(A - B)$.

Verify that

$$\det AB = \det A \det B$$

and that

$$\det(A - B)(A + B) = \det(A_B) \det(A + B)$$

Answer

$$AB = \begin{pmatrix} 7 & -6 \\ -11 & 10 \end{pmatrix}$$

$$BA = \begin{pmatrix} -2 & 3 \\ -14 & 19 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 16 & -21 \\ -28 & 37 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} 1 & 0 \\ -3 & 4 \end{pmatrix}$$

$$A^2 - B^2 = \begin{pmatrix} 15 & -21 \\ -25 & 33 \end{pmatrix}$$

$$A - B = \begin{pmatrix} -3 & 3 \\ 1 & -3 \end{pmatrix}$$

$$A + B = \begin{pmatrix} -1 & 3 \\ 7 & -7 \end{pmatrix}$$

$$(A - B)(A + B) = \begin{pmatrix} 24 & -30 \\ -22 & 24 \end{pmatrix}$$

$$(A + B)(A - B) = \begin{pmatrix} 6 & -12 \\ -28 & 42 \end{pmatrix}$$

$\det A = -2 \quad \det B = -2 \quad \det AB = 4 = (-2)(-2)$
$\det(A - B) = 6 \quad \det(A + B) = -14 \quad \det(A - B)(A + B) = -84 = 6 \times (-14)$