

Question

Find the general solution of the following differential equations:

1. $y'' + 5y' = 0$
2. $y'' - 6y' + 9y = 0$ (*)
3. $y'' + 2y' - 8y = 0$ (*)
4. $y'' + 6y' + 13y = 0$ (*)

Answer

Auxiliary equation is: $m^2 + 5m = 0$

which has roots $m = -5, 0$

Hence general solution is: $y = Ae^{-5x} + Be^{0x} = Ae^{-5x} + B$

1. Auxiliary equation is: $m^2 - 6m + 9 = 0$
which has roots $m = 3, 3$.
Since the roots are repeated the general solution is:
 $y = Ae^{3x} + Bxe^{3x} = (A + Bx)e^{3x}$
2. Auxiliary equation is: $m^2 + 2m - 8 = 0$
which has roots $m = \frac{-2 \pm \sqrt{4 + 32}}{2} = -4, 2$
Hence general solution is: $y = Ae^{-4x} + Be^{2x}$
3. Auxiliary equation is: $m^2 + 6m + 13 = 0$
which has roots $m = \frac{-6 \pm \sqrt{36 - 52}}{2} = -3 + 2i, -3 - 2i$
Hence the general solution is:
 $y = Ae^{(-3+2i)x} + Be^{(-3-2i)x} = e^{-3x}(C \cos(2x) + D \sin(2x))$