Question

Determine if the functions in the following sets are linearly independent.

1. The set 1, x, x^2.

2. The set $\cos x$, $\sin x$ $(*)$

Answer

1. Consider the Wronskian for the functions 1, x, x^2.

$$W(x) = \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & 2x \\ 0 & 0 & 2 \end{vmatrix} = 2$$

   Hence since $W \not\equiv 0$ the functions are linearly independent.

2. Consider the Wronskian for the functions $\sin x$, $\cos x$.

$$W(x) = \begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix} = -\sin^2 x - \cos^2 x = -1$$

   Hence since $W \not\equiv 0$ the functions are linearly independent.