QUESTION

Using the table provided, calculate the continuous-time premiums of the European call and put options of questions 1 and 2 of exercises 4 if the underlying asset pays a continuoue dividend of $0.01Sdt$, where S is the asset price at time t.

ANSWER

Use formula with $D = 0.01$ in:

call

$$
\begin{aligned}
C(S,t) &= se^{-D(T-t)}N(d_1) - ke^{-r(T-t)}N(d_2) \\
d_1 &= \frac{\left[\log\left(\frac{S}{k}\right) + (r - D + \frac{1}{2}\sigma^2)(T-t)\right]}{\sigma\sqrt{T-t}} \\
d_1 &= \frac{\left[\log\left(\frac{S}{k}\right) + (r - D - \frac{1}{2}\sigma^2)(T-t)\right]}{\sigma\sqrt{T-t}}
\end{aligned}
$$

Therefore at $t = 0$, initial premium given by

$$
\begin{aligned}
d_1 &= \frac{\left[\log\left(\frac{40}{50}\right) + \left(0.05 - 0.01 + \frac{0.3^2}{2}\right)\right]}{0.3} = -0.4605 \\
d_2 &= \frac{\left[\log\left(\frac{40}{50}\right) + \left(0.05 - 0.01 - \frac{0.3^2}{2}\right)\right]}{0.3} = -0.7605
\end{aligned}
$$

$N(-0.46) = 0.3228$
$N(-0.76) = 0.2236$
Therefore

$$
\begin{aligned}
C(40,0) &= 40 \times e^{-0.01} \times (0.3228) - 50e^{-0.05} \times 0.2236 \\
&= 12.7835 - 10.6347 \\
&= 2.1488
\end{aligned}
$$

which is less than the value of the option without a continuous dividend (since asset price will be falling by $DSdt$).

Put

$$
\begin{aligned}
P(S,t) &= -se^{-D(T-t)}N(-d_1) + ke^{-r(T-t)}N(-d_2) \\
&= -40 \times e^{-0.01}N(+0.46) + 50e^{-0.05}N(+0.76) \\
&= -40 \times e^{-0.01} \times 0.6772 + 50 \times e^{-0.05} \times 0.7764 \\
&= 10.1083
\end{aligned}
$$