QUESTION
Using the table provided, calculate the continuous-time premiums of the European call and put options of questions 1 and 2 of exercises 4. Compare the binomial and continuous answers.
ANSWER
$T = 1,\ k = \$50,\ r = 5\% = 0.05,\ \sigma = 30\% = 0.3,\ S_0 = \$40$ No dividend $\Rightarrow D = 0$.
Call:

$$\begin{aligned} C(S,t) &= se^{-D(T-t)}N(d_1) - ke^{-r(T-t)}N(d_2) \\ d_1 &= \frac{\left[\log\left(\frac{S}{k}\right) + (r - D + \frac{1}{2}\sigma^2)(T-t)\right]}{\sigma\sqrt{T-t}} \\ d_1 &= \frac{\left[\log\left(\frac{S}{k}\right) + (r - D - \frac{1}{2}\sigma^2)(T-t)\right]}{\sigma\sqrt{T-t}} \end{aligned}$$

Plug in numbers: at $t = 0$ for initial premium (NB $\log = \log_e$)

$$\begin{aligned} d_1 &= \frac{\left[\log\left(\frac{40}{50}\right) + \left(0.05 + \frac{0.3^2}{2}\right)\right]}{0.3} = -0.4271 \\ d_2 &= \frac{\left[\log\left(\frac{40}{50}\right) + \left(0.05 - \frac{0.3^2}{2}\right)\right]}{0.3} = -0.7271 \end{aligned}$$

Look up in table: Table only works to 2 d.p. so look up $N(-0.43) = 0.3336$ and $N(-0.73) = 0.2327$
Therefore $C(S_0, 0) = 40 \times 0.3336 - 50 \times e^{-0.5} \times 0.2327 = 2.2764$
Binomial value is 2.7261.
Put
$P(S,t) = -se^{-D(T-t)}N(-d_1) + ke^{-r(T-t)}N(-d_2)$
d_1 and d_2 are the same as above.

$$\begin{aligned} P(S_0,0) &= -40 \times N(+0.43) + 50e^{-0.05}N(0.73) \\ &= -40 \times 0.6664 + 50e^{-0.05} \times 0.7673 \\ &= 9.8379 \end{aligned}$$

Binomial value is 10.28750.