**Question**

a) Find the real and imaginary parts of the function $\sinh z$, where $z = x + iy$. Find the images of the lines $x$=constant and $y$= constant under the transformation $w = \sinh z$, identifying what kinds of curves they are.

b) Show that the points $z = 1$, $z = -\frac{1}{2}$ are inverse points with respect to the circle $C_1$ with centre $z = -1$ and radius 1.

Denote the circle with centre $z = +1$ and radius 1 by $C_2$.

Find a Mobius transformation

$$w = \frac{az + b}{cz + d}$$

which maps $z = 1$ to $w = -1$, the circle $C_1$ to the line $Re(w) = \frac{1}{2}$, and the circle $C_2$ to the line $Re(w) = -\frac{1}{2}$.

**Answer**

i) $\sin(x + iy) = \sinh x \cos y + i \cosh x \sin y = u + iv$

Therefore $u = \sinh x \cos y \qquad v = \cosh x \sin y$

So $x$=constant gives parametric equations for ellipses.

$y$=constant gives parametric equations for hyperbolas.

b) DIAGRAM

Now $A_1B_1 = \frac{1}{2}$, $A_1A_2 = 2$, $A_10 = 1$ therefore $A_2$ and $B_1$ are inverse with respect to $C_1$.

Similarly $A_1$ and $B_2$ are inverse with respect to $C_2$.

$C_1$ maps to $L_1$ so $A_2, B_1$ map to image points in $L_1$.

$A_2$ maps to $A_1$ so $B_1$ maps to $w = 2$.

$C_2$ maps to $L_2$ so $A_1, B_2$ map to image points in $L_2$.

$A_1$ maps to $A_2$ so $B_2$ maps to $w = -2$.

So we have

| $z$ | $w$ |
|---|---|
| $1$ | $-1$ |
| $-\frac{1}{2}$ | $2$ |
| $\frac{1}{2}$ | $-2$ |

So since $czw + dw - az - b = 0$, we have

$$
\begin{aligned}
-c - d - a - b &= 0 \quad (1) \\
-c + 2d + \frac{1}{2}a - b &= 0 \quad (2) \\
-c - 2d - \frac{1}{2}a - b &= 0 \quad (3)
\end{aligned}
$$

add 2 and 3, so $c + b = 0$, then 1 and 2 give $a + d = 0$, $2d + \frac{1}{2}a = 0$ giving $a = d = 0$. So the transformation is

$$w = -\frac{1}{z}$$