QUESTION Find the Laurent expansions of the following functions which converge in the regions indicated.

(a) $z^m e^{\frac{1}{z^2}},\ 0 < |z| < \infty$

(b) $\frac{1}{(z-1)(z+2)},\ 0 < |z-1| < 3$

ANSWER

(a)

$$z^m e^{\frac{1}{z^2}} = z^m \sum_{n=0}^{\infty} \frac{z^{-2n}}{n!} = \sum_{n=0}^{\infty} \frac{z^{m-2n}}{n!}$$

(using the exponential series)

(b)

$$\begin{aligned}
\frac{1}{(z-1)(z+2)} &= \frac{1}{z-1} \cdot \frac{1}{(z-1)+3} \\
&= \frac{1}{z-1} \cdot \frac{1}{3} \cdot \frac{1}{1+(\frac{z-1}{3})} \\
&= \frac{1}{3} \cdot \frac{1}{z-1} \sum_{n=0}^{\infty} \left(-\frac{z-1}{3}\right)^n \quad \text{(geometric series)} \\
&= \sum_{m=-1}^{\infty} (-1)^{m+1} 3^{-(m+2)} (z-1)^m \\
& \quad \text{(Taking } m = n-1\text{)}
\end{aligned}$$