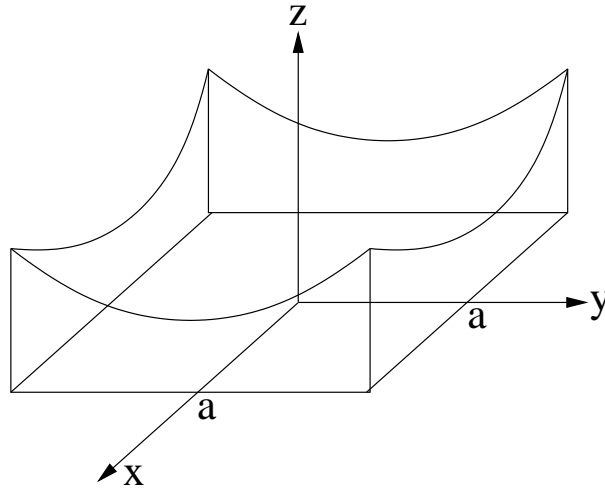**Question**

Sketch the region bounded by $z = x^2 + y^2$, $z = 0$, $x = -a$, $x = a$, $y = -a$, $y = a$, (where a is some unspecified constant) and calculate its volume by evaluating a suitable double integral.

Answer

Volume of region $= \int_{y=-a}^{y=a} \left\{ \int_{x=-a}^{x=a} (x^2 + y^2) \, dx \right\} dy$

$$= \int_{-a}^{a} \left[\frac{x^3}{3} + y^2 x \right]_{x=-a}^{x=a} dy = 2 \int_{-a}^{a} \frac{a^3}{3} + ay^2 \, dy$$

$$= 2 \left[\frac{a^3 y}{3} + \frac{ay^3}{3} \right]_{-a}^{a} = \frac{8a^4}{3}$$