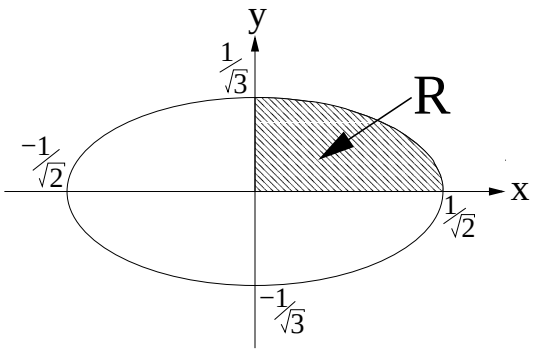
Question

Evaluate the double integral

$$\iint_R y\, d(x,y)$$

where R is the first quadrant of the ellipse $2x^2 + 3y^2 = 1$.

Answer

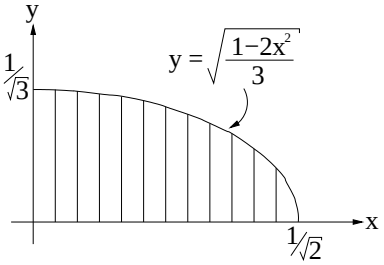


$2x^2 + 3y^2 = 1$

We will write the integral so that the integration with respect to y is performed first:

Take vertical lines(xfixed) so the region is defined by

$$0 \le y \le \sqrt{\tfrac{1-2x^2}{3}}$$
$$0 \le x \le \tfrac{1}{\sqrt{2}}$$



The integral becomes

$$\begin{aligned}
\iint_R y\, d(x,y) &= \int_{x=0}^{x=\frac{1}{\sqrt{2}}} \left\{ \int_{y=0}^{y=\sqrt{\frac{1-2x^2}{3}}} y\, dy \right\} dx \\
&= \int_0^{\frac{1}{\sqrt{2}}} \left[\frac{y^2}{2} \right]_{y=0}^{y=\sqrt{\frac{1-2x^2}{3}}} dx \\
&= \int_0^{\frac{1}{\sqrt{2}}} \frac{1-2x^2}{6}\, dx \\
&= \left[\frac{x}{6} - \frac{x^3}{9} \right]_0^{\frac{1}{\sqrt{2}}}
\end{aligned}$$

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\begin{aligned}
&= \frac{1}{6\sqrt{2}} - \frac{1}{18\sqrt{2}} \\
&= \frac{1}{9\sqrt{2}}
\end{aligned}
$$