

QUESTION

A and B are two independent events. A is twice as likely to occur as B and three times as likely to occur as the event that neither A or B does. Find the probability of A.

ANSWER

A and B are independent therefore $P(A \cap B) = P(A)P(B)$

A is twice as likely as B therefore $P(A) = 2P(B)$

A is three times as likely as neither A or B therefore $P(A) = 3P(\overline{A} \cap \overline{B})$

$P(\overline{A} \cap \overline{B})1 - P(A \cup B) = 1 - P(A) - P(B) - P(A \cap B)$

$$\begin{aligned}\frac{1}{3}P(A) &= 1 - P(A) - \frac{1}{2}P(A) + \frac{1}{2}[P(A)]^2 \\ &= 1 - \frac{3}{2}P(A) + \frac{1}{2}[P(A)]^2\end{aligned}$$

$$3[P(A)]^2 - 11P(A) + 6 = 0$$

$$(3P(A) - 2)(P(A) - 3) = 0$$

$P(A) = \frac{2}{3}$ or $P(A) = 3$. Since $0 \leq P(A) \leq 1$ $P(A) = \frac{2}{3}$