

QUESTION

Two events A and B are such that the probability of B given A is 4 times the probability of A and the probability of A given B is 9 times the probability of B. If the probability that at least one of A and B occurs is $\frac{7}{48}$, Find the probability of A.

ANSWER

$$P(B|A) = 4P(A) \rightarrow \frac{P(A \cap B)}{P(A)} = 4P(A)$$

$$P(A|B) = 9P(B) \rightarrow \frac{P(A \cap B)}{P(B)} = 9P(B)$$

$$\text{therefore } P(A \cap B) = 4[P(A)]^2 = 9[P(B)]^2$$

$$\text{therefore } 2P(A) = 3P(B)$$

$$\begin{aligned} \frac{7}{48} = P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= P(A) + \frac{2}{3}P(A) - 4[P(A)]^2 \end{aligned}$$

$$192[P(A)]^2 - 80P(A) + 7 = 0(8P(A) - 1)(24P(A) - 7) = 0$$

$P(A) = \frac{1}{8}$ or $P(A) = \frac{7}{24}$. Since $P(A) = \frac{7}{24} > \frac{7}{48} = P(A \cup B)$ this is not a possible solution. Therefore $P(A) = \frac{1}{8}$