Question

The **minimum value property** states that, if f is continuous on $[a, b]$, then f achieves its minimum on $[a, b]$; that is, there exists some y_0 in $[a, b]$ so that $f(y_0) \leq f(x)$ for all $x \in [a, b]$. Prove that a continuous function $f : [a, b] \to \mathbf{R}$ satisfies the minimum value property if it satisfies the maximum value property.

Answer

Since f is continuous on $[a, b]$, so is $g(x) = -f(x)$. Since g is continuous on the closed interval $[a, b]$, the maximum value property applied to g yields that there exists some x_0 in $[a, b]$ so that $g(x_0) \geq g(x)$ for all x in $[a, b]$. Hence, $-f(x_0) \geq -f(x)$ for all x in $[a, b]$, and so $f(x_0) \leq f(x)$ for all x in $[a, b]$. That is, f satisfies the minimum value property.