

Question

Suppose $\{f_n\}$ is a sequence of functions each of which is finite a.e. Show that, for almost all x in \mathbf{R}^n , $f_n(x)$ is finite for all n .

Answer

Let $A_n = \{x | f_n(x) = +\infty \vee f_n(x) = -\infty\}$

Then $m(A_n) = 0$. Let $S = \{x | \text{for all } n \in \mathbf{N} \quad -\infty < f_n(x) < \infty\}$

Then $S = \mathbf{R}^n - \bigcup_{n=1}^{\infty} A_n$

Therefore $m(S) = m(\mathbf{R}^n) - m\left(\bigcup_{n=1}^{\infty} A_n\right) = m(\mathbf{R}^n) - 0 = m(\mathbf{R}^n)$ and $m(S) = 0$