Question

Show that if $f : \mathbf{R^n} \to \mathbf{R}$ is measurable, so is $\dfrac{1}{f}$ (where $\dfrac{1}{0} = +\infty$).

Answer

$$\{x|\frac{1}{f}(x) < c\} = \begin{cases} \{x|\frac{1}{c} < f(x) < 0\} & \text{if} \quad c < 0 \\ \{x| -\infty < f(x) < 0\} & \text{if} \quad c = 0 \\ \{x| -\infty \leq f(x) < 0\} \cup \{x|f(x) > \frac{1}{c}\} & \text{if} \quad c > 0 \end{cases}$$