Question

If a projectile is fired due east from a point on the surface of the earth at a northern latitude λ with a speed V_0 and at an angle of inclination to the horizontal of α, show that the lateral deflection when the particle strikes the earth is

$$d = \frac{4V_0^3}{g}\omega \sin\lambda \sin^2\alpha \cos\alpha.$$

Answer

Same set up as 2b.

Initial conditions: $\dot{y} = V_0 \cos\alpha, \dot{z} = V_0 \sin\alpha; \quad x = y = z = 0.$

First approximation: set $\omega = 0$

$y = (V_0 \cos\alpha)t$

$z = (V_0 \sin\alpha)t - \frac{1}{2}gt^2$

Returns to ground when $t = \frac{2V_0 \sin\alpha}{g}(*)$

Next approximation:

$$\left.\begin{array}{rcl} \ddot{x} &=& 2\omega \sin\lambda V_0 \cos\alpha \\ \ddot{y} &=& -2\omega \cos\lambda(V_0 \sin\alpha - gt) \\ \ddot{z} &=& 2\omega \cos\lambda V_0 \cos\alpha - g \end{array}\right\} \text{From N2 as in 2b.}$$

Thus $x = V_0\omega \sin\lambda \cos\alpha t^2$

Putting this in (*) gives $x = V_0\omega \sin\lambda \cos\alpha \frac{4v_0^2 \sin^2\alpha}{g^2}$ (as required)

i.e. projectile deflected south by this amount.